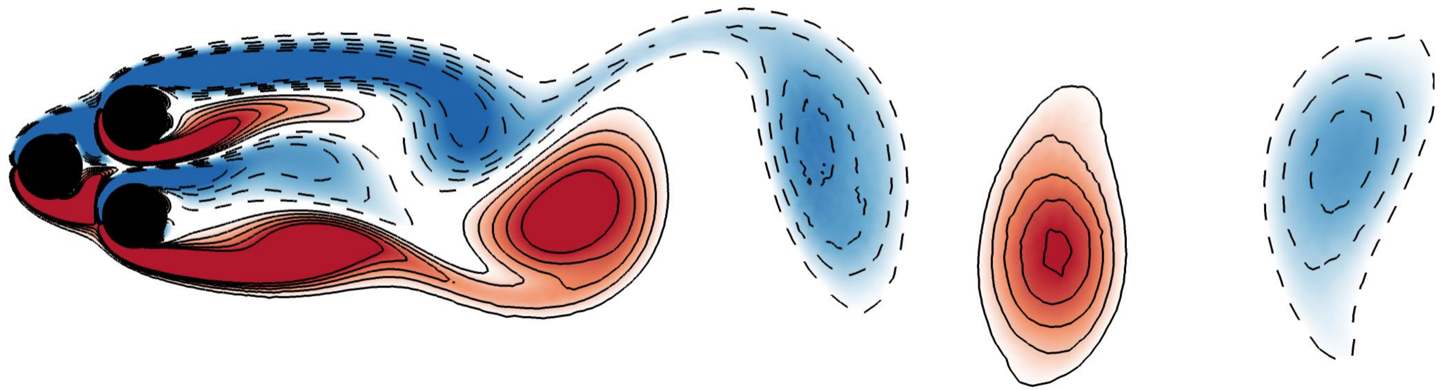

Controlling the fluidic Pinball with machine and human learning

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collaborated with: Guy CORNEJO-MACEDA [2], François LUSSEYRAN [2]

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[2] LIMSI – CNRS, Université Paris Saclay, Orsay, France
[3] Harbin Institute of Technology, China
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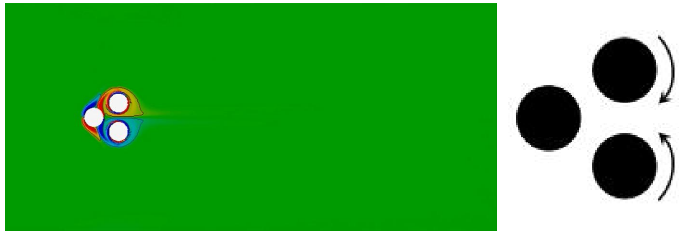
[6] Poznan University of Technology, Pologne

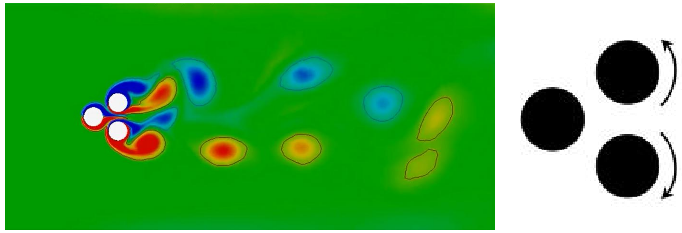
Fluidic Pinball

MIMO (Multiple Input Multiple Output) system

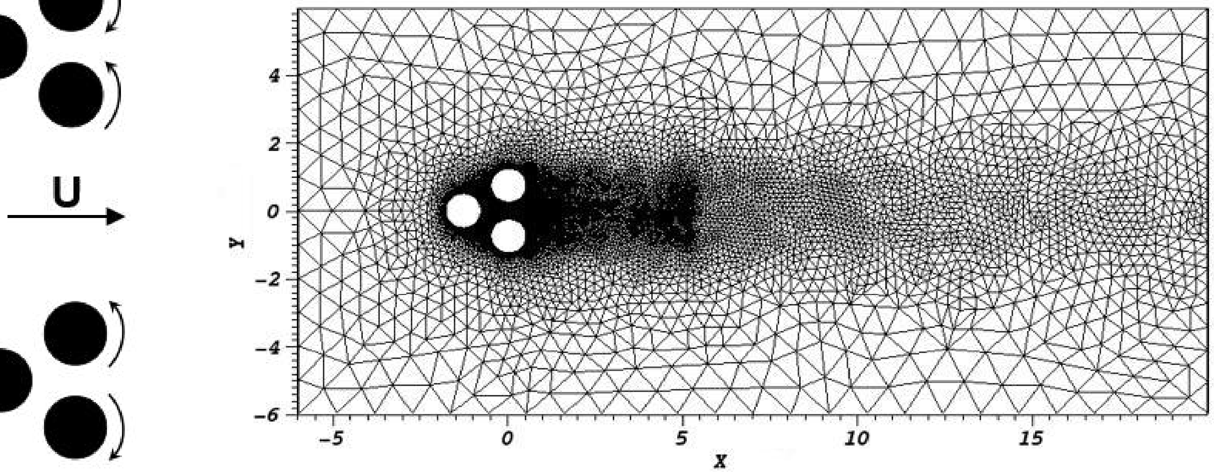
- Geometrically simple configuration & fast computation.
- Physically rich enough to comprise many dynamical regimes and to allow testing many control laws.

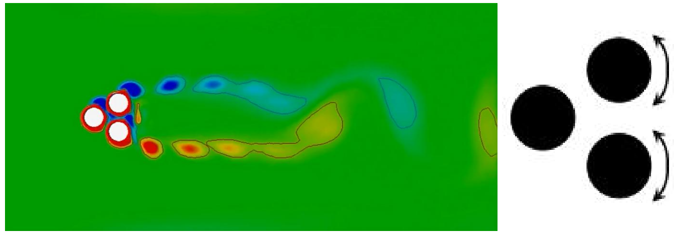


Boat tailing (Coanda)

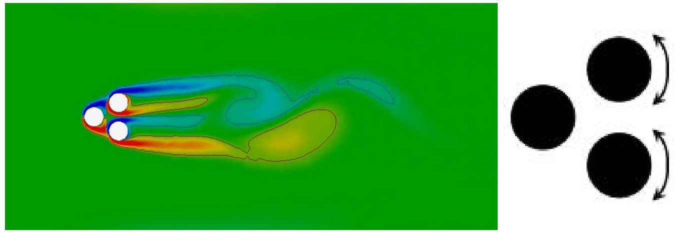


Base bleed

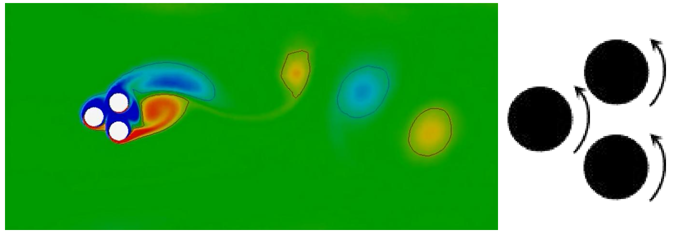




High-frequency control



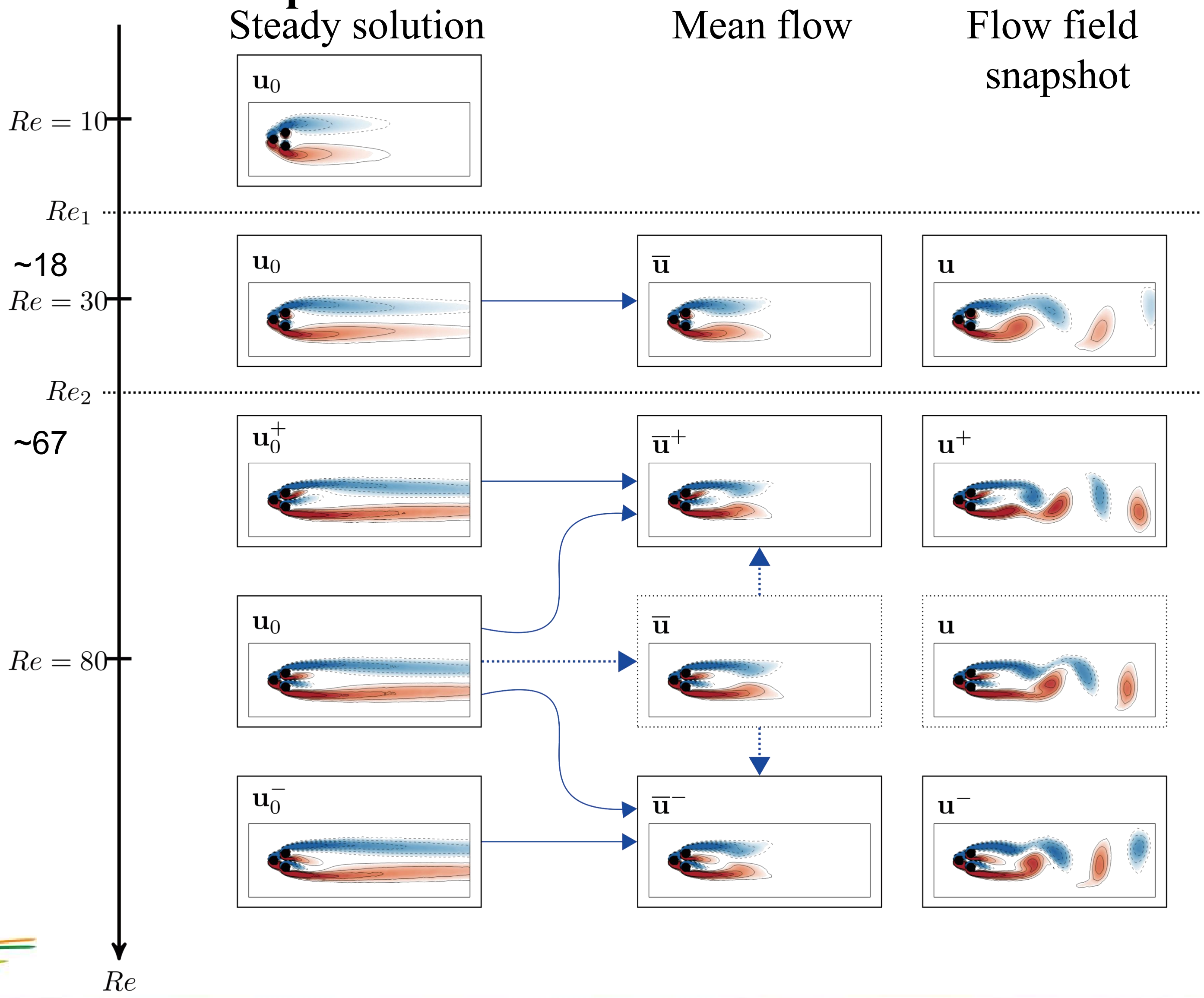
Low-frequency control



Stagnation point control (Magnus effect)

Unforced Fluidic Pinball

Reynolds-number dependent flow behavior



Route to chaos:

Hopf
+
Pitchfork
+
Neimark-Sacker
↓
Chaos

Newhouse-Ruelle-Takens route to chaos.

Unforced Fluidic Pinball

Galerkin method + Mean-field modeling

$$\vec{u}(\vec{x},t) = \vec{u}_0(\vec{x}) + \sum_{i=1}^{N} a_i(t)\vec{u}_i(\vec{x})$$

$$\partial_t \vec{u} + \nabla \cdot \vec{u} \otimes \vec{u} = \nu \Delta \vec{u} - \nabla p$$

Linear-quadratic Garlerkin system

$$\frac{d}{dt} a_i = \sum_{j=1}^{N} l_{ij} a_j + \sum_{j,k=1}^{N} q_{ijk} a_j a_k$$

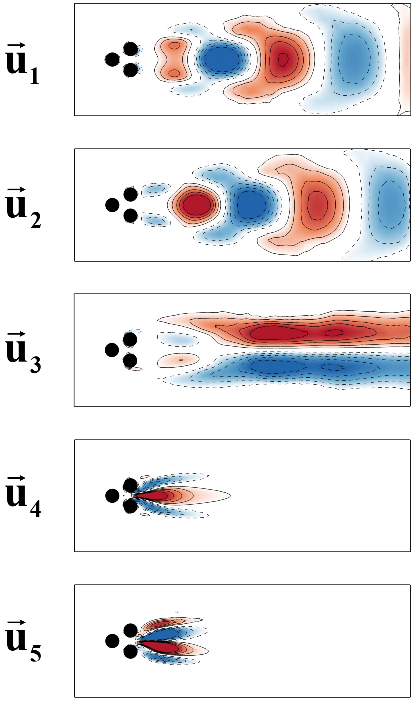
With a series physical constraints:

Hopf

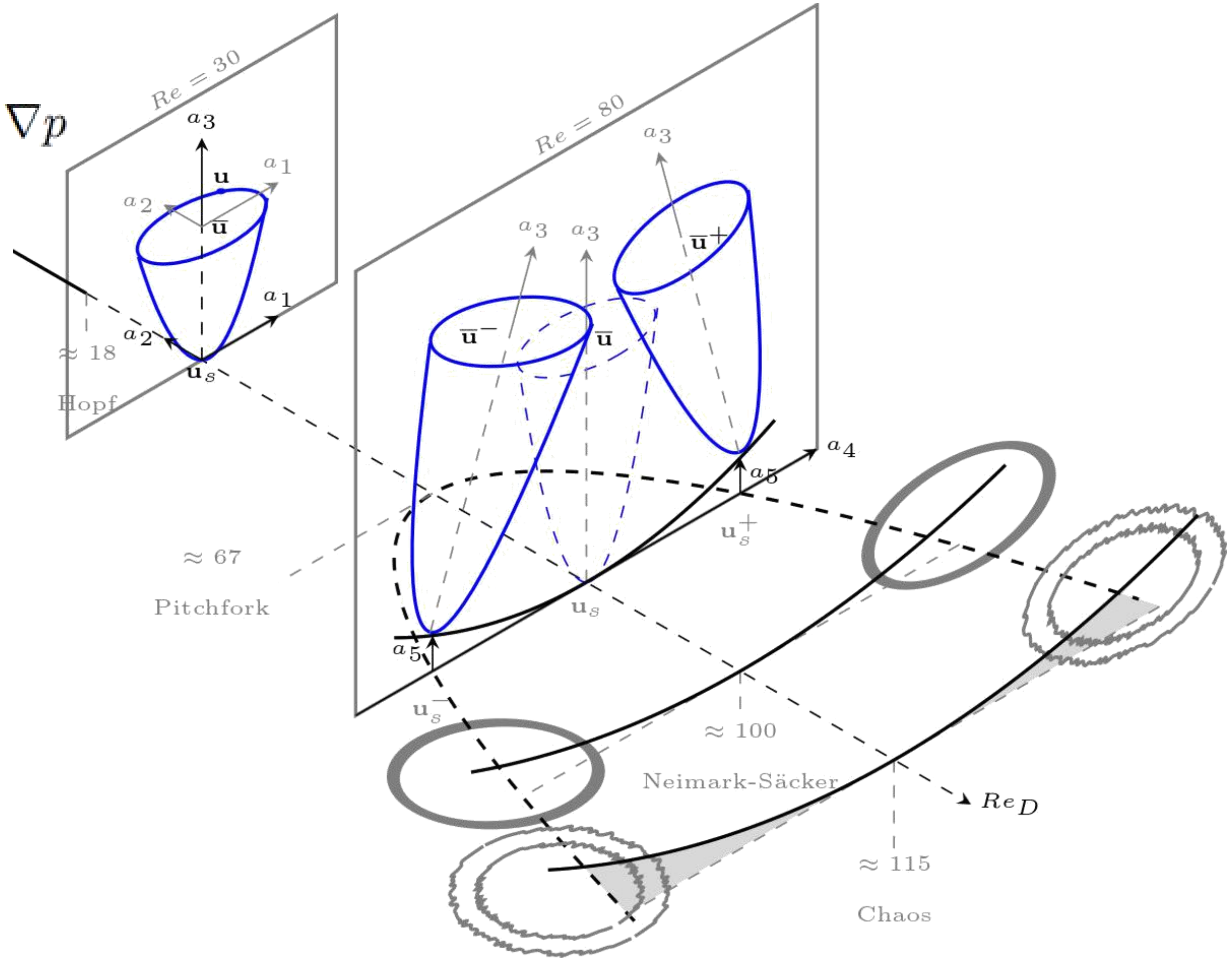
$$da_1/dt = \sigma a_1 - \omega a_2$$
$$da_2/dt = \sigma a_2 + \omega a_1$$
$$da_3/dt = \sigma_3 a_3 + \beta_3 r^2$$

+ Pitchfork

$$da_4/dt = \sigma_4 a_4 - \beta_4 a_4 a_5$$
$$da_5/dt = \sigma_5 a_5 + \beta_5 a_4^2$$



Bifurcation diagram



Unforced Fluidic Pinball

Galerkin method + Mean-field modeling

$$\vec{u}(\vec{x},t) = \vec{u}_0(\vec{x}) + \sum_{i=1}^{N} a_i(t)\vec{u}_i(\vec{x})$$

$$\partial_t \vec{u} + \nabla \cdot \vec{u} \otimes \vec{u} = \nu \Delta \vec{u} - \nabla p$$

Linear-quadratic Garlerkin system

$$\frac{d}{dt}a_i = \sum_{j=1}^{N} l_{ij}a_j + \sum_{j,k=1}^{N} q_{ijk}a_j a_k$$

With a series physical constraints:

Hopf

$$da_1/dt = \sigma a_1 - \omega a_2$$

$$da_2/dt = \sigma a_2 + \omega a_1$$

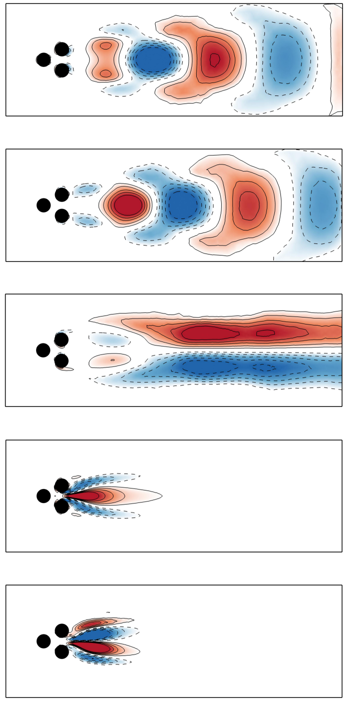
$$da_3/dt = \sigma_3 a_3 + \beta_3 r^2$$

+ Pitchfork

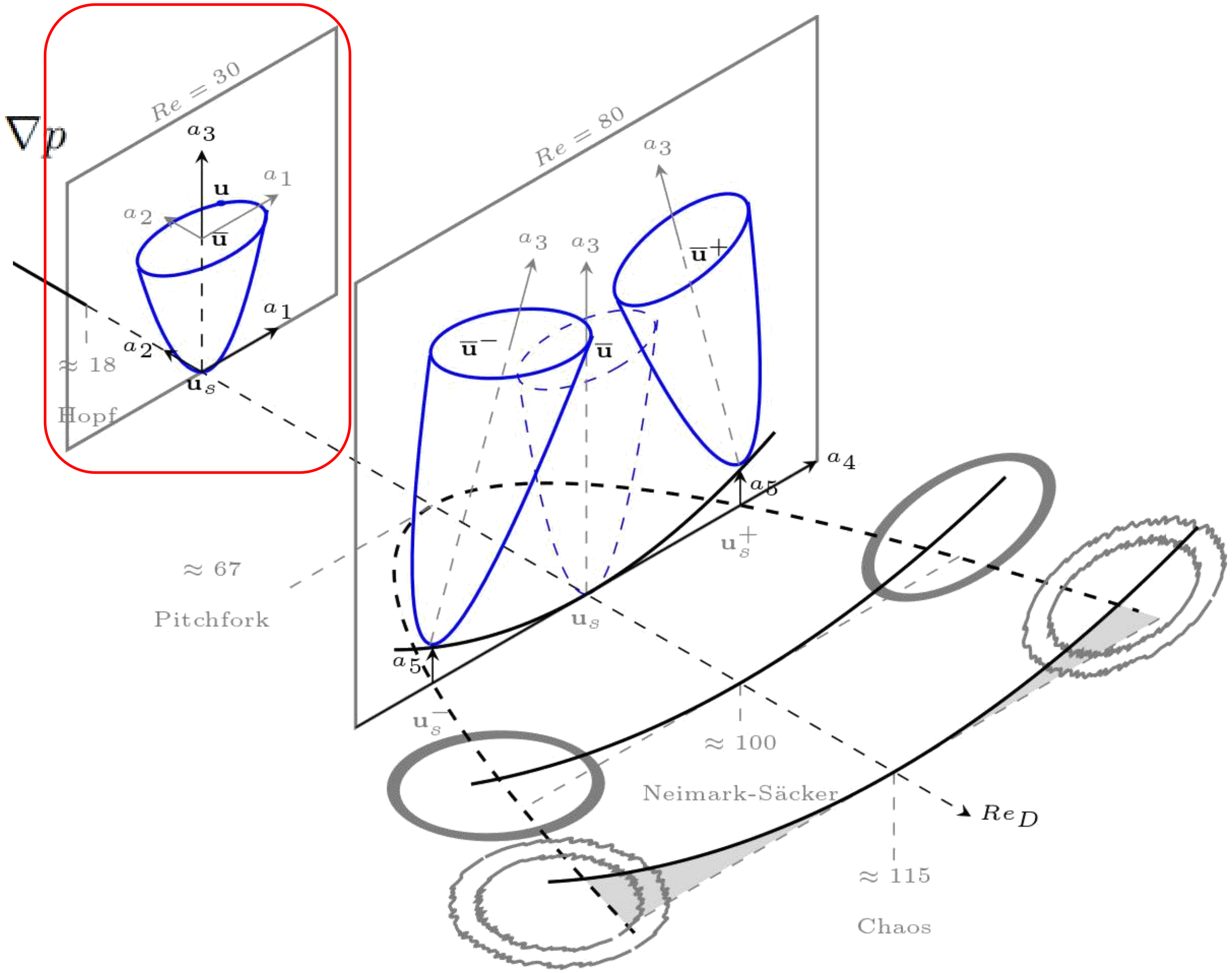
$$da_4/dt = \sigma_4 a_4 - \beta_4 a_4 a_5$$

$$da_5/dt = \sigma_5 a_5 + \beta_5 a_4^2$$

$\vec{\mathbf{u}}_1$, $\vec{\mathbf{u}}_2$, $\vec{\mathbf{u}}_3$, $\vec{\mathbf{u}}_4$, $\vec{\mathbf{u}}_5$



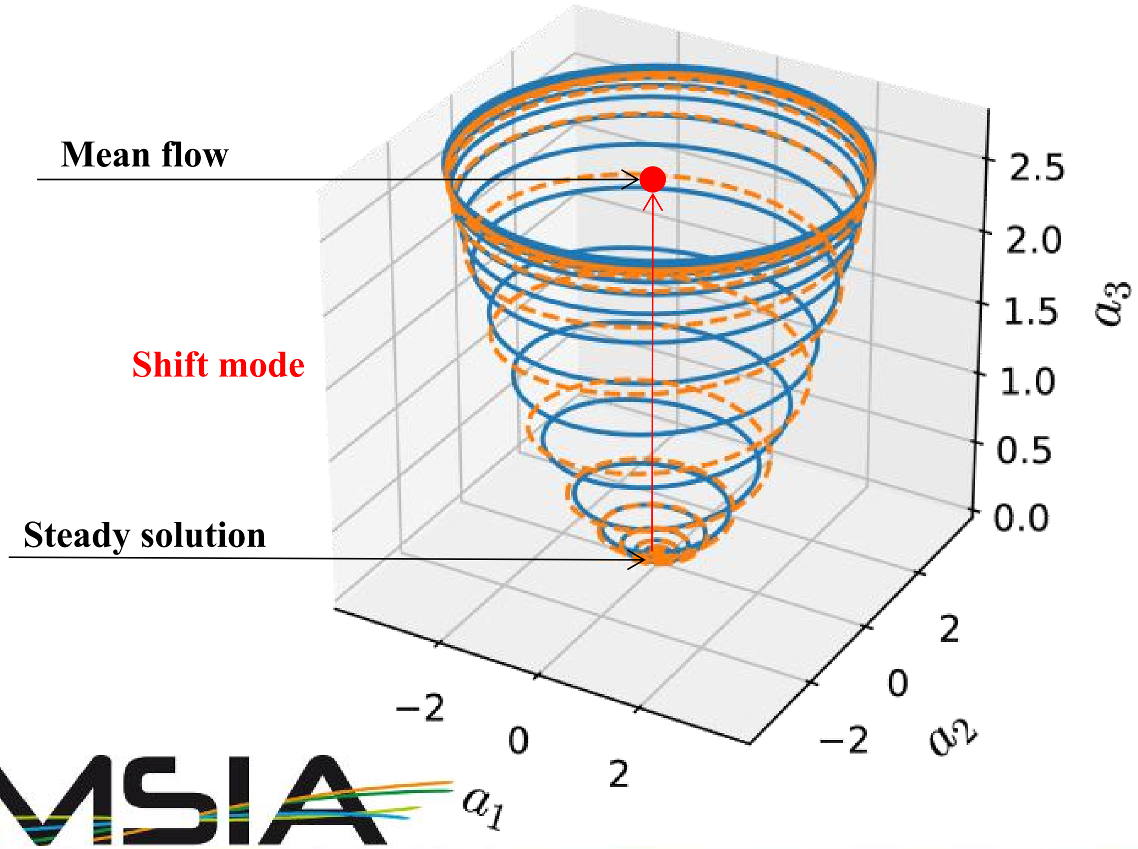
Bifurcation diagram

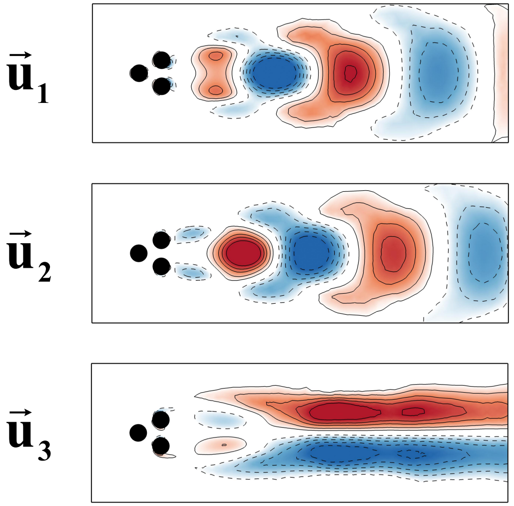


Example 1

For the primary flow regime (Re = 30)

$$\mathbf{u}(\mathbf{r},t) \simeq \underbrace{\mathbf{u}_s(\mathbf{r})}_{\text{steady solution}} \underbrace{+\, a_1(t)\mathbf{u}_1(\mathbf{r}) + a_2(t)\mathbf{u}_2(\mathbf{r})}_{\text{leading POD modes}} \underbrace{+\, a_3(t)\mathbf{u}_3(\mathbf{r})}_{\text{shift mode}}$$





Generalized mean field system with 3 d.o.f. :

$$da_1/dt = \sigma a_1 - \omega a_2, \qquad \sigma = \sigma_1 - \beta a_3$$
$$da_2/dt = \sigma a_2 + \omega a_1, \qquad \omega = \omega_1 + \gamma a_3$$
$$da_3/dt = \sigma_3 a_3 + \beta_3 \left(a_1^2 + a_2^2\right)$$

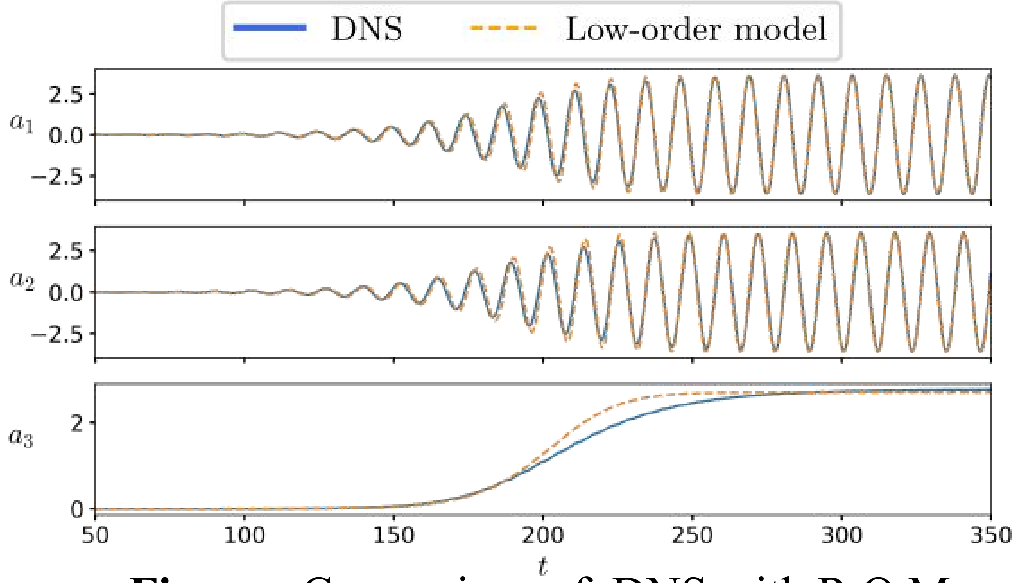


Figure : Comparison of DNS with R.O.M.

Example 2

For the primary flow regime (Re = 80)

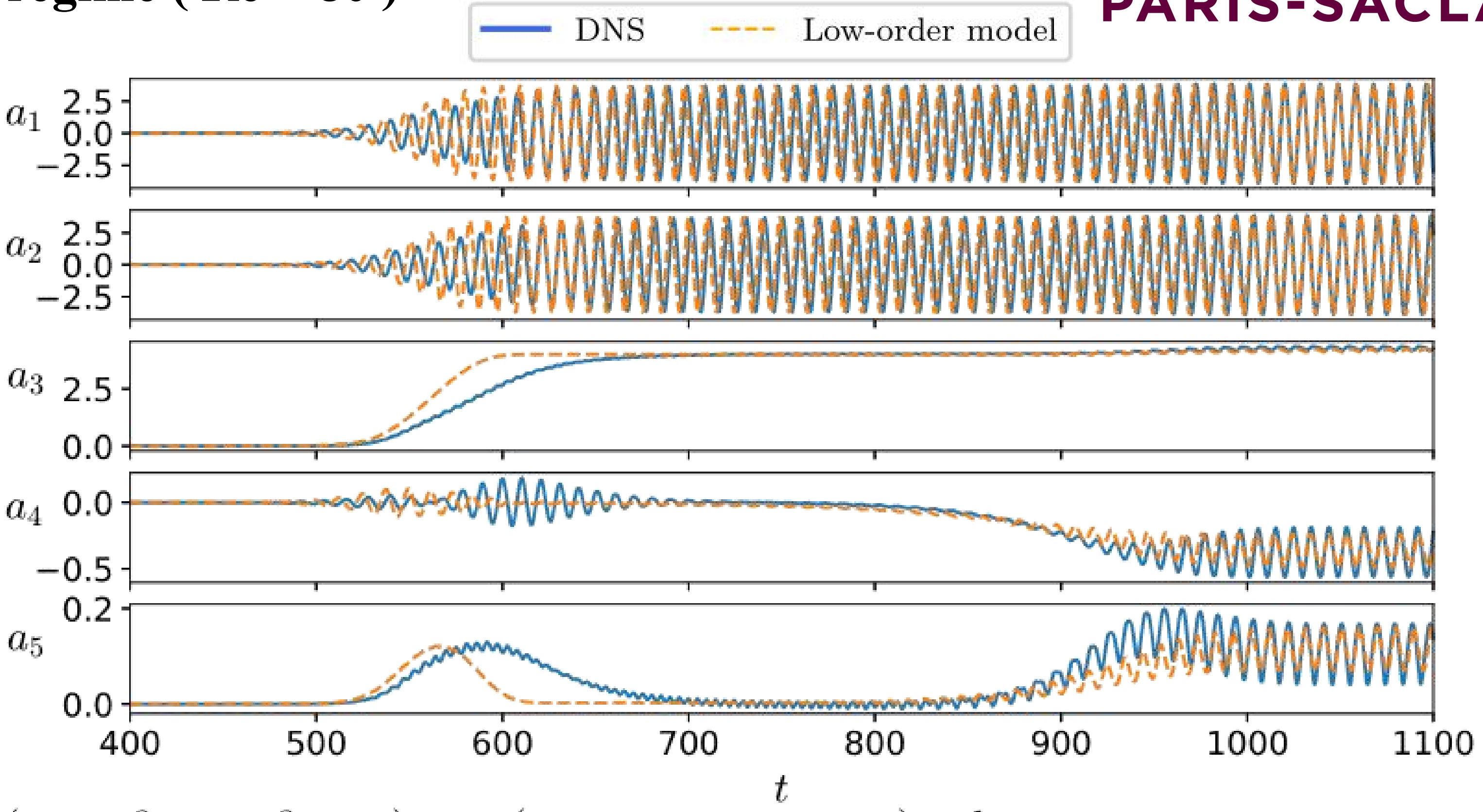
Hopf

$$\dot{a_1} = \sigma(a_3)a_1 - \omega(a_3)a_2,$$
$$\dot{a_2} = \sigma(a_3)a_2 + \omega(a_3)a_1,$$
$$\dot{a_3} = \sigma_3 a_3 + \beta_3(a_1^2 + a_2^2),$$

PF

$$\dot{a_4} = \sigma_4 a_4 - \beta_4 a_4 a_5,$$
$$\dot{a_5} = \sigma_5 a_5 + \beta_5 a_4^2,$$

Cross terms



The cross terms are identified under a sparsity constraint with the SINDy algorithm (Brunton et al. 2016).

$$da_1/dt = a_1(\sigma_1 - \beta\ a_3 - \beta_{15}\ a_5) - a_2(\omega_1 + \gamma\ a_3 + \gamma_{15}\ a_5) + l_{14}\ a_4 + q_{134}\ a_3 a_4,$$
$$da_2/dt = a_2(\sigma_1 - \beta\ a_3 - \beta_{15}\ a_5) + a_1(\omega_1 + \gamma\ a_3 + \gamma_{15}\ a_5) + l_{24}\ a_4 + q_{234}\ a_3 a_4,$$
$$da_3/dt = \sigma_3\ a_3 + \beta_3\ r^2 + l_{35}\ a_5 + q_{314}\ a_1 a_4 + q_{335}\ a_3 a_5 + q_{355}\ a_5^2,$$
$$da_4/dt = \sigma_4\ a_4 - \beta_4\ a_4 a_5 + a_1(l_{41} + q_{413}\ a_3 + q_{415}\ a_5) + a_2(l_{42} + q_{423}\ a_3 + q_{425}\ a_5),$$
$$da_5/dt = \sigma_5\ a_5 + \beta_5\ a_4^2 + l_{53}\ a_3 + q_{514}\ a_1 a_4 + q_{533}\ a_3^2 + q_{535}\ a_3 a_5.$$

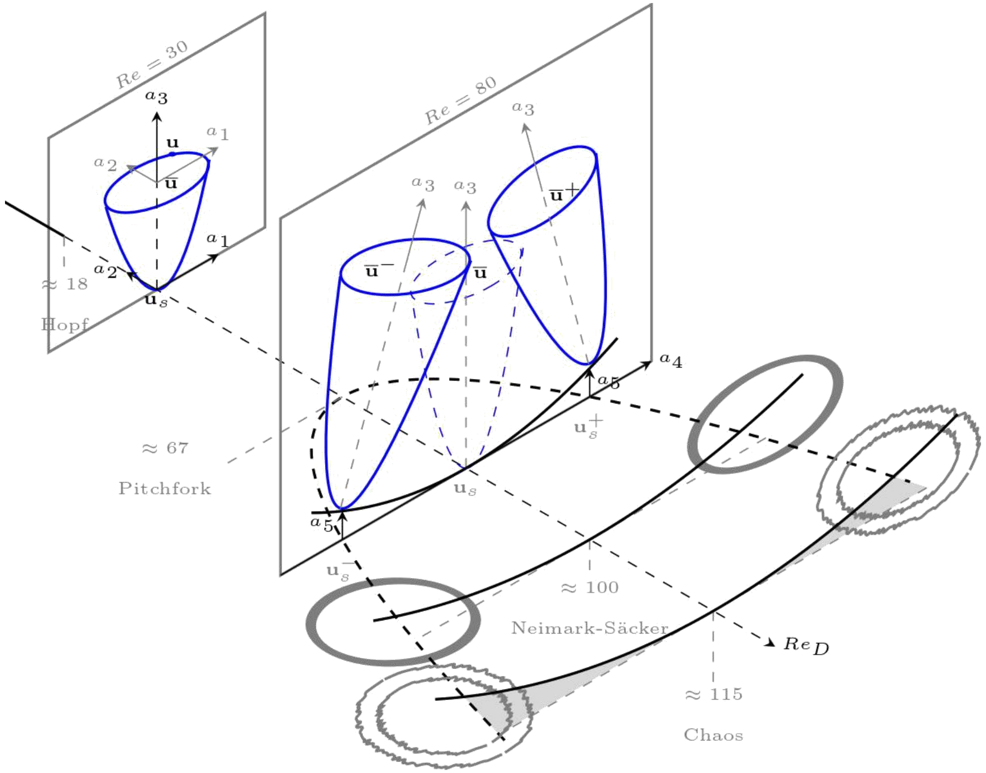
Conclusions

Controlling the fluidic Pinball with machine and human learning

- ✓ How the unforced fluidic pinball goes to chaos
- ✓ How to get R. O. M. for successive bifurcations

Perspectives:

- ➢ R.O.M. for quasi-periodic regime and chaos?
- ➢ Other bifurcations in the forced fluidic pinball?
- ➢ Re dependence of modes & coefficients?
- ➢ Automatization:
  Human learning to machine learning?



Low-order model for successive bifurcations of the pinball fluidique.
arXiv preprint arXiv :1812.08529 (2018)
J. Fluid Mech., 2018. (In revision)

Thank you for your attention.

Any questions?

ANNEX 1 -- R. O. M. of Fluidic Pinball

Ansatz & methodology

Galerkin method

Rempfer & Fasel (1994)

$$\vec{u}(\vec{x},t) = \vec{u}_0(\vec{x}) + \sum_{i=1}^{N} a_i(t)\vec{u}_i(\vec{x})$$

$$\partial_t \vec{u} + \nabla \cdot \vec{u} \otimes \vec{u} = \nu \Delta \vec{u} - \nabla p$$

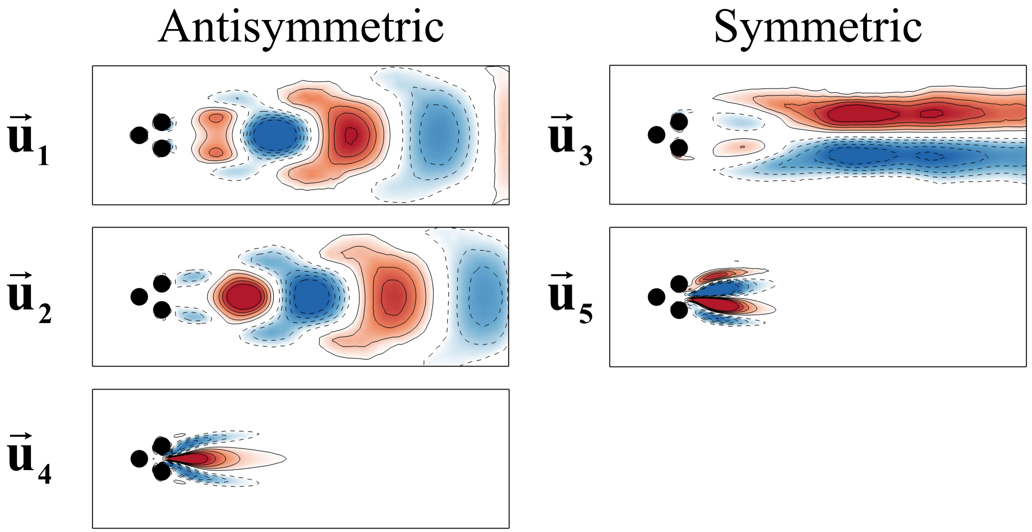
$$\frac{d}{dt}a_i = \nu \sum_{j=0}^{N} l_{ij}^{\nu} a_j + \sum_{j,k=0}^{N} q_{ijk}^{c} a_j a_k$$

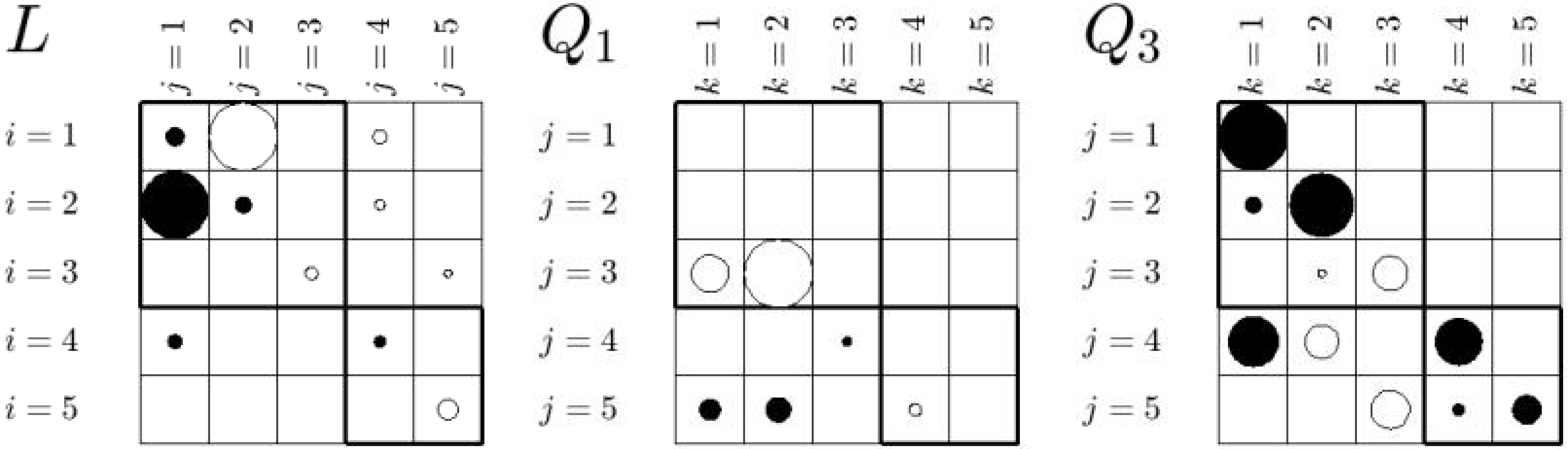
$$\vec{u}_0 = \vec{u}_s$$

Linear-quadratic Garlerkin system

$$\frac{d}{dt}a_i = \sum_{j=1}^{N} l_{ij} a_j + \sum_{j,k=1}^{N} q_{ijk} a_j a_k$$

Mean-field modeling





ANNEX 1 -- R. O. M. of Fluidic Pinball

Ansatz & methodology

Galerkin method

$$\vec{u}(\vec{x},t) = \vec{u}_s(\vec{x}) + \underbrace{a_1(t)\vec{u}_1(\vec{x}) + a_2(t)\vec{u}_2(\vec{x})}_{\vec{u}'} + \underbrace{a_3\vec{u}_3(\vec{x})}_{\vec{u}_\Delta}$$

Supercritical Hopf

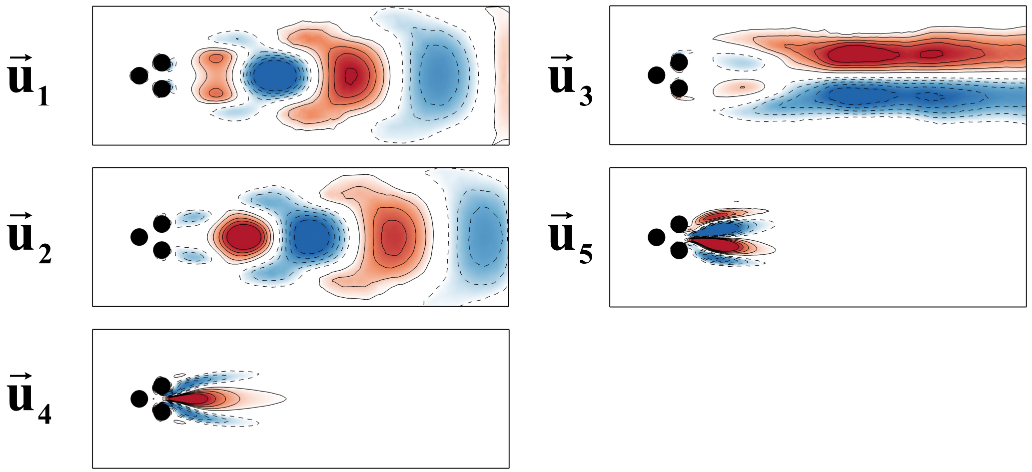
$$\vec{u}(\vec{x},t) = \vec{u}_s(\vec{x}) + \underbrace{a_4(t)\vec{u}_4(\vec{x})}_{\vec{u}'} + \underbrace{a_5(t)\vec{u}_5(\vec{x})}_{\vec{u}_\Delta}$$

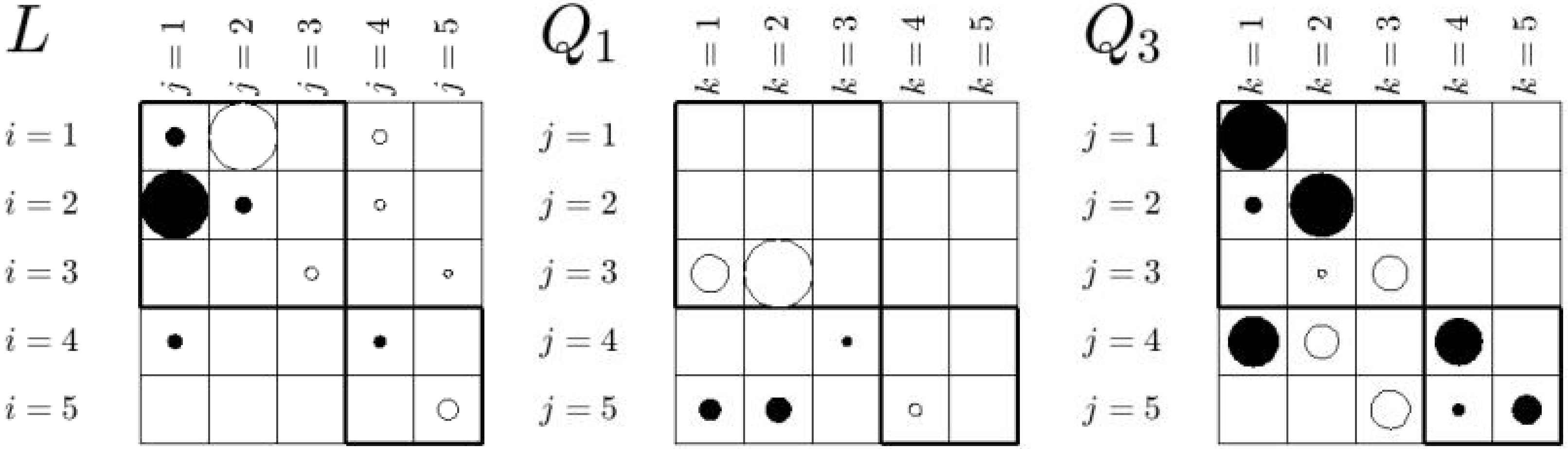
Supercritical Pitchfork

$$\partial_t \vec{u} + \nabla \cdot \vec{u} \otimes \vec{u} = \nu \Delta \vec{u} - \nabla p$$

$$\frac{d}{dt} a_i = \sum_{j=1}^{N} l_{ij} a_j + \sum_{j,k=1}^{N} q_{ijk} a_j a_k$$

Mean-field modeling





ANNEX 1 -- R. O. M. of Fluidic Pinball

Ansatz & methodology

Galerkin method

$$\vec{u}(\vec{x},t) = \vec{u}_s(\vec{x}) + \underbrace{a_1(t)\vec{u}_1(\vec{x}) + a_2(t)\vec{u}_2(\vec{x})}_{\vec{u}'} + \underbrace{a_3\vec{u}_3(\vec{x})}_{\vec{u}_\Delta}$$

Supercritical Hopf

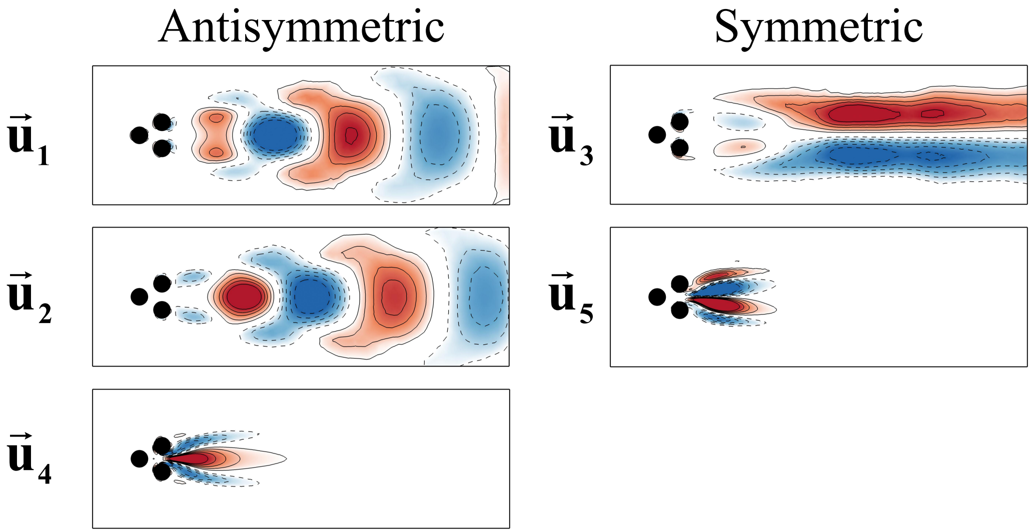
$$\vec{u}(\vec{x},t) = \vec{u}_s(\vec{x}) + \underbrace{a_4(t)\vec{u}_4(\vec{x})}_{\vec{u}'} + \underbrace{a_5(t)\vec{u}_5(\vec{x})}_{\vec{u}_\Delta}$$

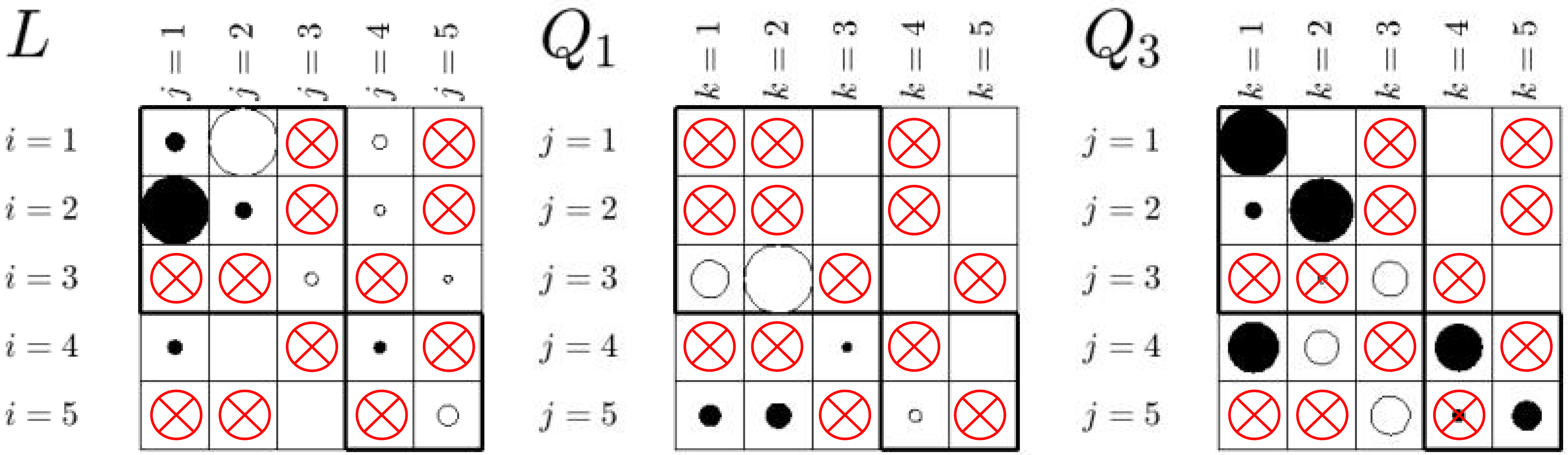
Supercritical Pitchfork

$$\partial_t \vec{u} + \nabla \cdot \vec{u} \otimes \vec{u} = \nu \triangle \vec{u} - \nabla p$$

Symmetry constraints

Mean-field modeling





ANNEX 1 -- R. O. M. of Fluidic Pinball

Ansatz & methodology

Galerkin method

$$\vec{u}(\vec{x},t) = \vec{u}_s(\vec{x}) + \underbrace{a_1(t)\vec{u}_1(\vec{x}) + a_2(t)\vec{u}_2(\vec{x})}_{\vec{u}'} + \underbrace{a_3\vec{u}_3(\vec{x})}_{\vec{u}_\Delta}$$

Supercritical Hopf

$$\vec{u}(\vec{x},t) = \vec{u}_s(\vec{x}) + \underbrace{a_4(t)\vec{u}_4(\vec{x})}_{\vec{u}'} + \underbrace{a_5(t)\vec{u}_5(\vec{x})}_{\vec{u}_\Delta}$$

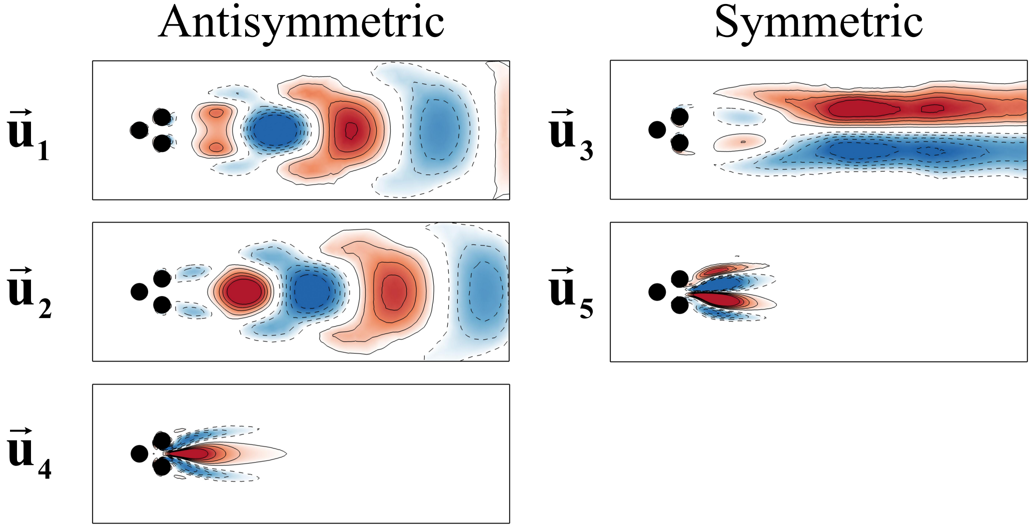
Supercritical Pitchfork

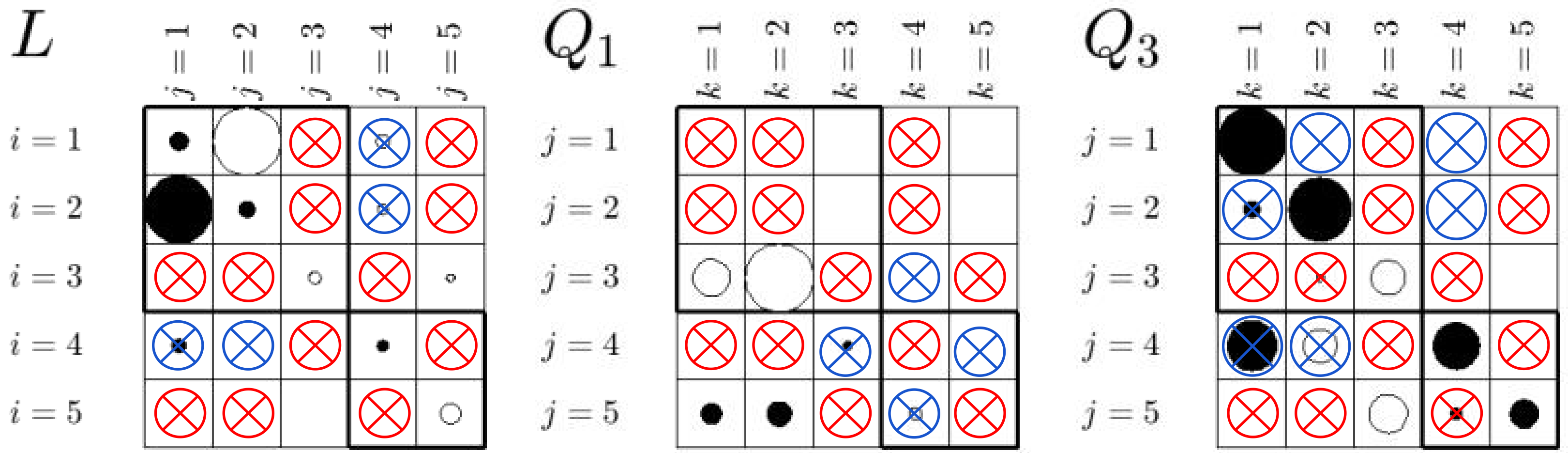
Symmetry constraints

Kryloff-Bogoliulov approximation

$$\partial_t \vec{u} + \nabla \cdot \vec{u} \otimes \vec{u} = \nu \triangle \vec{u} - \nabla p$$

Mean-field modeling





ANNEX 1 -- R. O. M. of Fluidic Pinball

Ansatz & methodology

Galerkin method

$$\vec{u}(\vec{x},t) = \vec{u}_s(\vec{x}) + \underbrace{a_1(t)\vec{u}_1(\vec{x}) + a_2(t)\vec{u}_2(\vec{x})}_{\vec{u}'} + \underbrace{a_3\vec{u}_3(\vec{x})}_{\vec{u}_\Delta}$$

Supercritical Hopf

$$\vec{u}(\vec{x},t) = \vec{u}_s(\vec{x}) + \underbrace{a_4(t)\vec{u}_4(\vec{x})}_{\vec{u}'} + \underbrace{a_5(t)\vec{u}_5(\vec{x})}_{\vec{u}_\Delta}$$

Supercritical Pitchfork

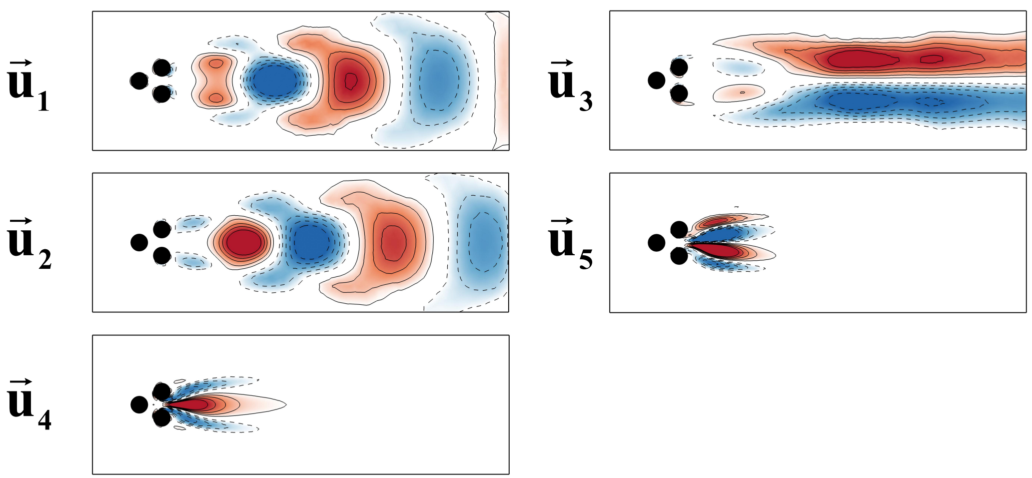
Symmetry constraints

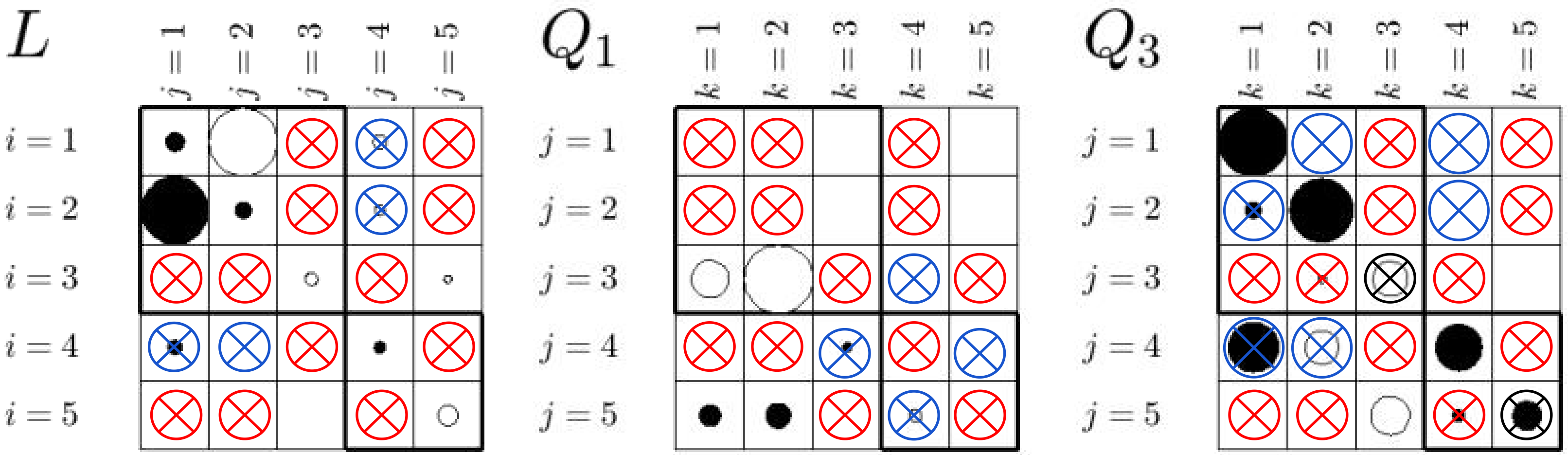
$$\partial_t \vec{u} + \nabla \cdot \vec{u} \otimes \vec{u} = \nu \Delta \vec{u} - \nabla p$$

Kryloff-Bogoliulov approximation

$\vec{u}_\Delta \in O(\delta)$,$O(\delta^2)$ can be neglected

Mean-field modeling





ANNEX 1 -- R. O. M. of Fluidic Pinball

Ansatz & methodology

Galerkin method

$$\vec{u}(\vec{x},t) = \vec{u}_s(\vec{x}) + \underbrace{a_1(t)\vec{u}_1(\vec{x}) + a_2(t)\vec{u}_2(\vec{x})}_{\vec{u}'} + \underbrace{a_3\vec{u}_3(\vec{x})}_{\vec{u}_\Delta}$$

Supercritical Hopf

$$\vec{u}(\vec{x},t) = \vec{u}_s(\vec{x}) + \underbrace{a_4(t)\vec{u}_4(\vec{x})}_{\vec{u}'} + \underbrace{a_5(t)\vec{u}_5(\vec{x})}_{\vec{u}_\Delta}$$

Supercritical Pitchfork

Symmetry constraints

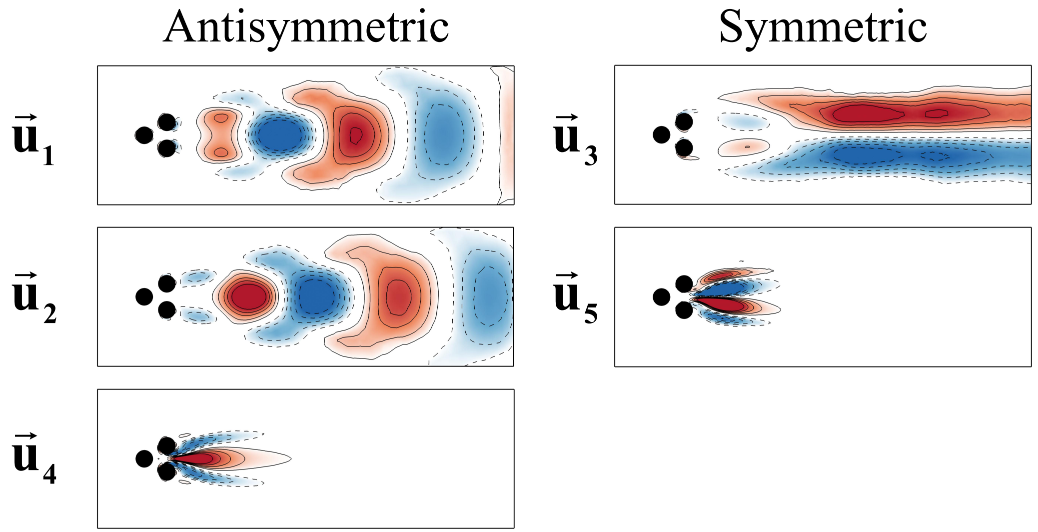
$$\partial_t \vec{u} + \nabla \cdot \vec{u} \otimes \vec{u} = \nu \Delta \vec{u} - \nabla p$$

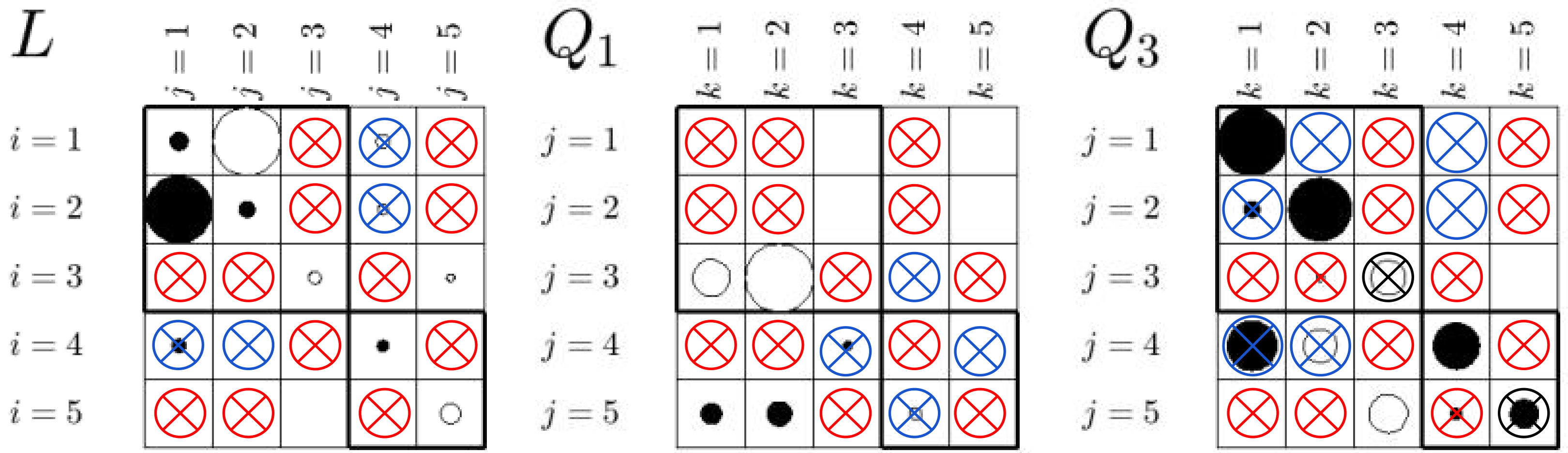
Kryloff-Bogoliulov approximation

$\vec{u}_\Delta \in O(\delta)$,$O(\delta^2)$ can be neglected

$$da_1/dt = \sigma a_1 - \omega a_2, \qquad \sigma = \sigma_1 - \beta a_3$$
$$da_2/dt = \sigma a_2 + \omega a_1, \qquad \omega = \omega_1 + \gamma a_3$$
$$da_3/dt = \sigma_3 a_3 + \beta_3 \left(a_1^2 + a_2^2\right)$$
$$da_4/dt = \sigma_4 a_4 - \beta_4 a_4 a_5$$
$$da_5/dt = \sigma_5 a_5 + \beta_5 a_4^2$$

Mean-field modeling

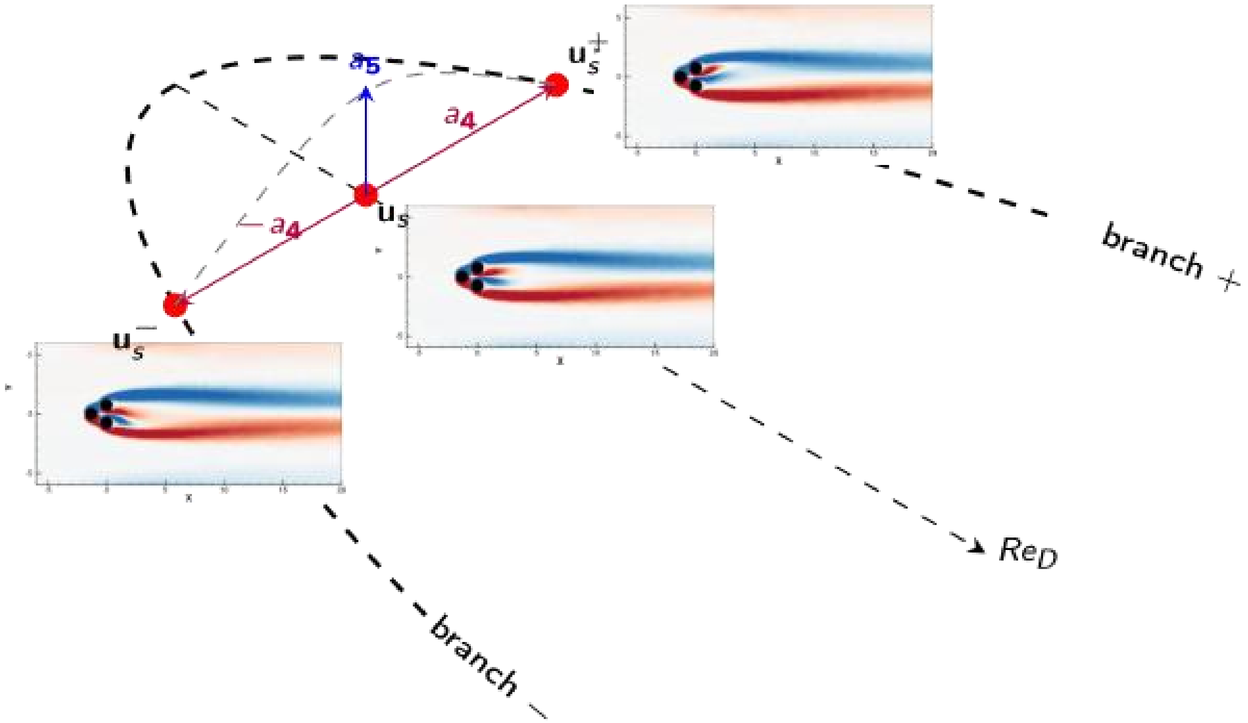




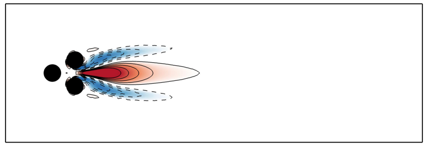
ANNEX 2 -- R. O. M. of Fluidic Pinball

For the secondary flow regime (Re = 80)

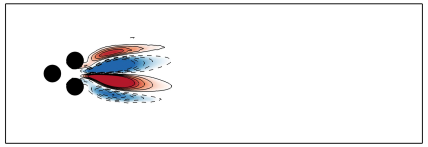
$$\mathbf{u}(\mathbf{r},t) \simeq \underbrace{\mathbf{u}_s(\mathbf{r})}_{\text{steady solution}} + \underbrace{a_1(t)\mathbf{u}_1(\mathbf{r}) + a_2(t)\mathbf{u}_2(\mathbf{r})}_{\text{leading POD modes}} + \underbrace{a_3(t)\mathbf{u}_3(\mathbf{r})}_{\text{shift mode}} + \underbrace{a_4(t)\mathbf{u}_4(\mathbf{r}) + a_5(t)\mathbf{u}_5(\mathbf{r})}_{\text{Pitch-Fork bifurcation}}$$



$\vec{\mathbf{u}}_4$



$\vec{\mathbf{u}}_5$

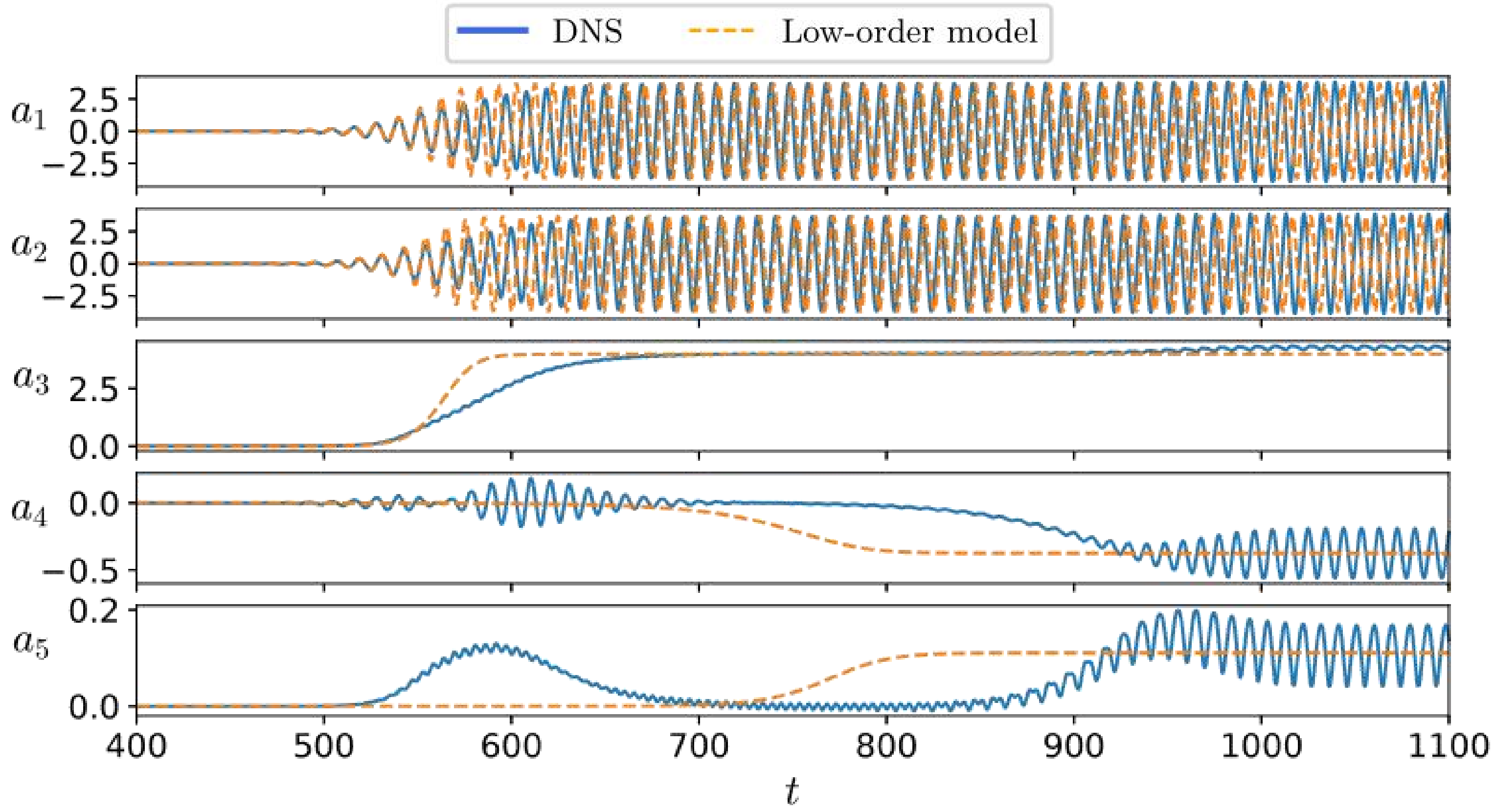


$$\mathbf{u}_4 = \frac{1}{2}(\mathbf{u}^+{}_s - \mathbf{u}^-{}_s)$$

$$\mathbf{u}_5 = \frac{1}{2}(\mathbf{u}^+{}_s + \mathbf{u}^-{}_s) - \mathbf{u}_s$$

ANNEX 2 -- R. O. M. of Fluidic Pinball

R. O. M. without cross-terms



Generalized mean field system with 5 d.o.f. :

$$
\begin{aligned}
da_1/dt &= \sigma a_1 - \omega a_2, \qquad & \sigma &= \sigma_1 - \beta a_3 \\
da_2/dt &= \sigma a_2 + \omega a_1, \qquad & \omega &= \omega_1 + \gamma a_3 \\
da_3/dt &= \sigma_3 a_3 + \beta_3 \left(a_1^2 + a_2^2\right) \\
da_4/dt &= \sigma_4 a_4 - \beta_4 a_4 a_5 \\
da_5/dt &= \sigma_5 a_5 + \beta_5 a_4^2
\end{aligned}
$$

List of coefficients :

σ_1	5.22×10^{-2}	β	1.31×10^{-2}
ω_1	5.24×10^{-1}	γ	2.95×10^{-2}
σ_3	-5.22×10^{-1}	β_3	1.53×10^{-1}
σ_4	2.72×10^{-2}	β_4	2.45×10^{-1}
σ_5	-2.72×10^{-1}	β_5	2.14×10^{-1}

The coefficients are identified from the linear stability analysis and the asymptotic dynamics.

ANNEX 2 -- R. O. M. of Fluidic Pinball

R. O. M. with cross-terms

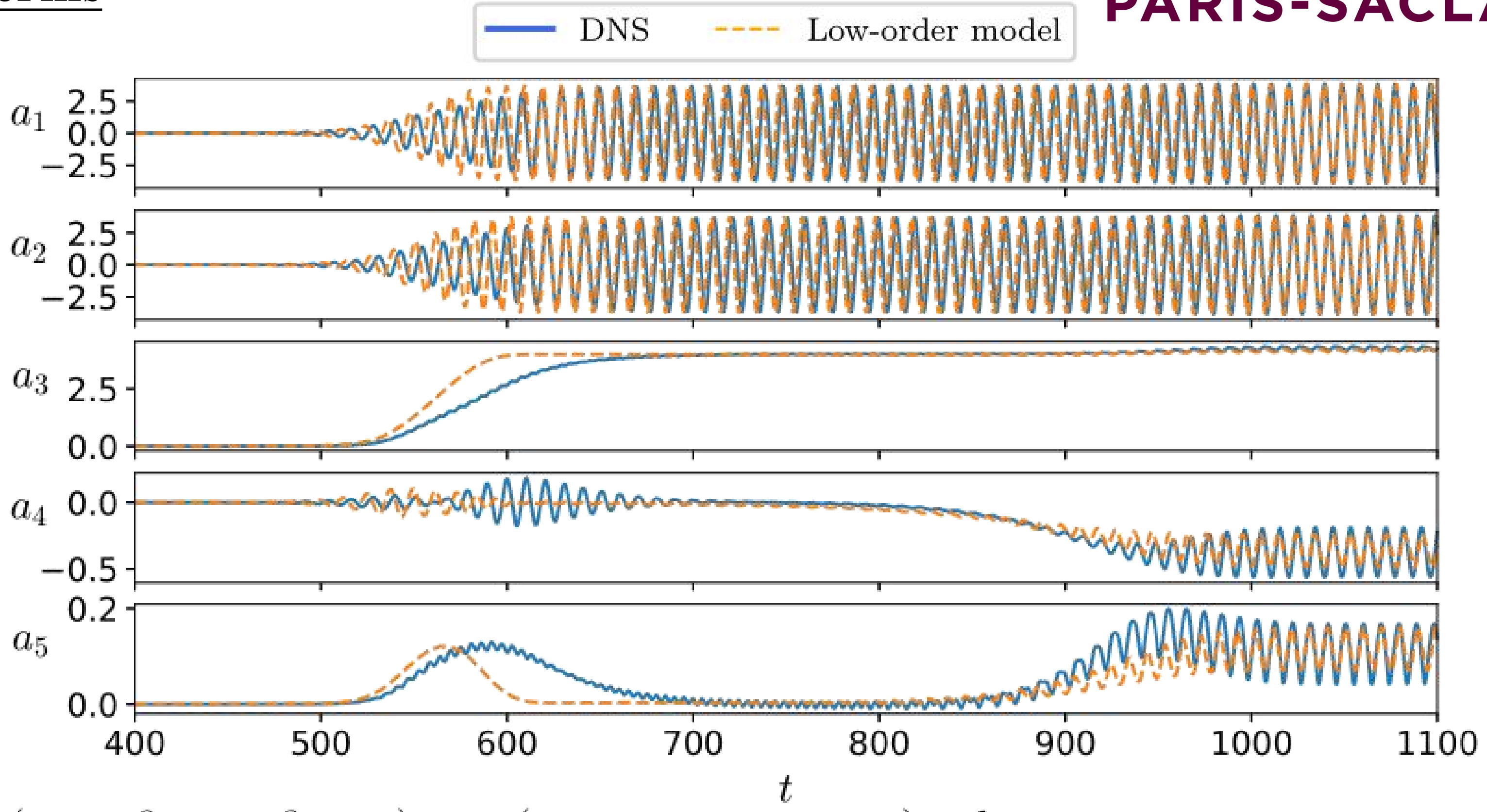
Hopf

$$\dot{a_1} = \sigma(a_3)a_1 - \omega(a_3)a_2,$$
$$\dot{a_2} = \sigma(a_3)a_2 + \omega(a_3)a_1,$$
$$\dot{a_3} = \sigma_3 a_3 + \beta_3(a_1^2 + a_2^2),$$

PF

$$\dot{a_4} = \sigma_4 a_4 - \beta_4 a_4 a_5,$$
$$\dot{a_5} = \sigma_5 a_5 + \beta_5 a_4^2,$$

Cross terms



The cross terms are identified under a sparsity constraint with the SINDy algorithm (Brunton et al. 2016).

$$da_1/dt = a_1(\sigma_1 - \beta\ a_3 - \beta_{15}\ a_5) - a_2(\omega_1 + \gamma\ a_3 + \gamma_{15}\ a_5) + l_{14}\ a_4 + q_{134}\ a_3 a_4,$$
$$da_2/dt = a_2(\sigma_1 - \beta\ a_3 - \beta_{15}\ a_5) + a_1(\omega_1 + \gamma\ a_3 + \gamma_{15}\ a_5) + l_{24}\ a_4 + q_{234}\ a_3 a_4,$$
$$da_3/dt = \sigma_3\ a_3 + \beta_3\ r^2 + l_{35}\ a_5 + q_{314}\ a_1 a_4 + q_{335}\ a_3 a_5 + q_{355}\ a_5^2,$$
$$da_4/dt = \sigma_4\ a_4 - \beta_4\ a_4 a_5 + a_1(l_{41} + q_{413}\ a_3 + q_{415}\ a_5) + a_2(l_{42} + q_{423}\ a_3 + q_{425}\ a_5),$$
$$da_5/dt = \sigma_5\ a_5 + \beta_5\ a_4^2 + l_{53}\ a_3 + q_{514}\ a_1 a_4 + q_{533}\ a_3^2 + q_{535}\ a_3 a_5.$$